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STABILITY OF PLANE-PARALLEL ELECTROHYDRODYNAMIC FLOWS IN A LONGITUDINAL ELECTRIC FIELD

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UDC 539.19

In recent years there have been significant developments in a new branch of electromagnetic hydrodynamics, electrohydrodynamics. Theoretical studies in this field have been stimulated by various technological applications [1-5]. Functioning electrohydrodynamic pumps, generators, and dc transformers have now been constructed. However, further development of electrohydrodynamic (EHD) devices is being hindered by the incomplete, and sometimes, even contradictory, nature of existing concepts involving the principles of electrophysics and the hydromechanics of weakly conducting liquids and gases [2]. One may say that the least-studied area involves questions of stability and turbulence of EHD flows. The present study will analyze the stability of plane-parallel to EHD flows in a longitudinal electric field with respect to small perturbations.

We choose as a characteristic length the quantity l_0, equal to one-half the channel width, and select as the characteristic velocity V_0. Let E_0 be the intensity of the externally applied electric field. We will measure electric field intensity, space charge density, time, pressure, and current density in units E_0, $\tilde{\rho}_e$, l_0/V_0, ρV_0^2, $K\tilde{\rho}_e E_0$ (where ρ is the liquid density and K is the ion mobility coefficient). Then the system of equations describing the EHD of a viscous incompressible fluid can be written in dimensionless form as

$$\partial\mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -\nabla p + (1/\mathrm{Re})\Delta\mathbf{v} + \mathrm{Eu}_\alpha\,\rho_e\mathbf{E} + \Gamma\nabla\mathbf{E}^2; \quad (1)$$

$$\operatorname{div}\mathbf{v} = 0; \quad (2)$$

$$\operatorname{rot}\mathbf{E} = 0; \quad (3)$$

$$\operatorname{div}\mathbf{E} = \mathrm{Re}_\alpha\,\rho_e; \quad (4)$$

$$\partial\rho_e/\partial t + (1/\mathrm{M}_\alpha)\operatorname{div}\mathbf{j} = 0; \quad (5)$$

$$\mathbf{j} = \rho_e(\mathrm{M}_\alpha\mathbf{v} + \mathbf{E}) - (1/\mathrm{Re}_i)\nabla\rho_e. \quad (6)$$

Here $\mathbf{v}$ is velocity; $\mathbf{E}$, electric field intensity; ρ_e, space charge density; $\mathbf{j}$, current density; Re, Reynolds number; $\mathrm{Re}_\alpha = \tilde{\rho}_e l_0/\varepsilon\varepsilon_0 E_0$, electrical Reynolds number; $\mathrm{M}_\alpha = V_0/KE_0$, electric Mach number; $\mathrm{Re}_i = Kl_0E_0/D$, ionic Reynolds number; $\mathrm{Eu}_\alpha = \tilde{\rho}_e E_0 l_0/\rho V_0^2$, electrical Euler number; $\Gamma = (\varepsilon - \varepsilon_0)E_0^2/2\rho V_0^2$, electrical pressure number (where ε is the dielectric permittivity of the fluid and D is the diffusion coefficient).

The Navier–Stokes equation (1) considers the effect of the electric field on the charged liquid (the term $\mathrm{Eu}_\alpha\rho\mathrm{E}$) and the force acting on the weakly polarized dielectric in an

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inhomogeneous electric field (the term $\Gamma\nabla\mathbf{E}^2$). The incompressibility condition gives Eq. (2). Equation (4) is one of the electrodynamics equations. In EHD effects produced by magnetic induction are neglected. The electric field is assumed to be a potential field, as expressed by Eq. (3). The law of conservation of charge is expressed by Eq. (5), while Eq. (6) expresses Ohm's law.

Ohm's law (6) considers the conduction current, convection current, and self-diffusion current. The electrical similarity criteria M_a and Re_i define the contributions of diffusion and convective components to the current density. If the quantity Re_i is large, then the diffusion component of the current may be neglected; if M_a is small, then the convective component may be neglected.

We note that EHD phenomena are rarely found in pure form. Magnetohydrodynamic (MHD) processes can be neglected when the characteristic electrical charge relaxation time is much larger than the characteristic magnetic diffusion time. The interactions are then electrohydrodynamic, and the magnetic induction is negligibly small, so that charge conservation effects are significant [1].

We will consider the steady-state motion of a charged fluid in a planar channel under the action of an external constant electrostatic field and a pressure gradient. We will assume that the electric field is directed along the x axis of a Cartesian coordinate system. The y axis is perpendicular to the channel walls. The wall coordinates are then $y = \pm 1$. We will assume that the quantity $\partial p/\partial x$ is constant. Then the expression for velocity can be written in the form [6]

$$U = u_1(1 - y^2) + u_2(1 - \ln\cos By/\ln\cos B), \tag{7}$$

$$u_1 = -\frac{\mathrm{Re}}{2}\frac{\partial p}{\partial x},\quad u_2 = 2\frac{\mathrm{Re}\,\mathrm{Eu}_a}{\mathrm{Re}_a\mathrm{Re}_i}\ln\left(\frac{1}{\cos B}\right),\quad B = \sqrt{\frac{\mathrm{Re}_i\mathrm{Re}_a}{2}\rho_{e0}}.$$

Here ρ_{eo} is the dimensionless charge density on the channel axis at $y = 0$. We choose for V_0 the highest flow velocity attained on the channel axis. Then Eq. (7) can be written as

$$U = \delta(1 - y^2) + (1 - \delta)(1 - \ln\cos By/\ln\cos B). \tag{8}$$

The value of the parameter δ varies from zero to unity depending on the ratio between the electrical and mechanical forces.

The value of the induced transverse electric field is given by the expression [6] $E_y = (2/Re_i)B \tan By$.

The charge density distribution over the channel section is given by [6] $\rho_e = \rho_{eo}/\cos^2 By$. Hence we find the mean charge density over the channel section

$$\bar{\rho}_e = \rho_{e0}\,\mathrm{tg}\,B/B.$$

It is evident from this expression that the parameter B may vary over a range from 0 to $\pi/2$ (and it is for this reason that the argument of the logarithm in Eqs. (7), (8) is always positive).

The case $\delta = 1$ corresponds to Poiseuille flow. At $\delta = 0$ the flow has a completely pondermotor nature, i.e., it is produced solely by electrical forces:

$$U = 1 - \ln\cos By/\ln\cos B. \tag{9}$$

It is interesting that at small B the electrical forces form a velocity profile practically identical to the velocity profile of Poiseuille flow. At B values close to $\pi/2$ $(B > 1.5)$, the velocity profile (9) in the central portion of the channel is smoothed, and near the channel walls the velocity vanishes rapidly.

We will analyze the stability of such a stationary flow with respect to small perturbations.

Linearizing Eqs. (1)-(6) with respect to small perturbations, we obtain the following system:

$$\frac{\partial\mathbf{v}'}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}' + (\mathbf{v}'\nabla)\mathbf{v} = -\nabla p' + \frac{1}{\mathrm{Re}}\Delta\mathbf{v}' + \mathrm{Eu}_a(\rho_a\mathbf{E}' + \mathbf{E}\rho_e') + 2\Gamma\nabla(\mathbf{E}\mathbf{E}'); \tag{10}$$

$$\mathrm{div}\,\mathbf{v}' = 0; \tag{11}$$

$$\mathrm{rot}\,\mathbf{E}' = 0; \tag{12}$$

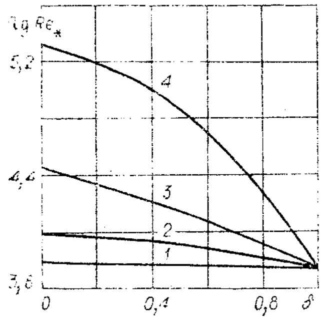

Fig. 1

$$\operatorname{div}\mathbf{E}' = \mathrm{Re}_a\,\rho'_e; \tag{13}$$

$$\frac{\partial\rho'_e}{\partial t} + \frac{1}{\mathrm{M}_a}\operatorname{div}\mathbf{j}' = 0; \tag{14}$$

$$\mathbf{j}' = \rho'_e(\mathrm{M}_a\mathbf{v} + \mathbf{E}) + \rho_e(\mathrm{M}_a\mathbf{v}' + \mathbf{E}') - \frac{1}{\mathrm{Re}_a}\nabla\rho'_e. \tag{15}$$

The prime denotes small perturbations here. We will seek a solution of system (10)-(15) in the form of elementary wave solutions.

From Eqs. (10), (11), using Eqs. (12), (13), we find the differential equation

$$L[v] \equiv \frac{1}{i\alpha\,\mathrm{Re}}\left(\frac{d^2}{dy^2} - k^2\right)^2 v - \left[(U - C)\left(\frac{d^2}{dy^2} - k^2\right)v - \frac{d^2U}{dy^2}v\right] = \tag{16}$$

$$= \frac{\mathrm{Eu}_a}{\alpha^2\,\mathrm{Re}_a}\left[-i\alpha\left(\frac{d^2}{dy^2} - k^2\right)\frac{dE'_x}{dy} - k^2E_y\left(\frac{d^2}{dy^2} - k^2\right)E'_x + k^2\frac{d^2E_y}{dy^2}E'_x\right].$$

From Eqs. (14), (15), using Eqs. (11)-(13), we obtain a second differential equation

$$\frac{1}{\mathrm{Re}_i}\left(\frac{d^2}{dy^2} - k^2\right)^2 E'_x - E_y\left(\frac{d^2}{dy^2} - k^2\right)\frac{dE'_x}{dy} - i\alpha\left[1 + M_a(U - c) + \frac{2}{i\alpha}\frac{dE_y}{dy}\right]\left(\frac{d^2}{dy^2} - k^2\right)E'_x - i\alpha\,\mathrm{M}_a\frac{d^2E_y}{dy^2}v - \frac{d^2E_y}{dy^2}\frac{dE'_x}{dy} = 0.$$

Equations (16), (17) compose a system of eighth-order complex equations in the complex amplitude of the y-component of the velocity perturbation v and the complex amplitude of the x-component of the electric field perturbation E'_x. We note that the dimensionless parameter Γ, which describes the action of electrical forces on a weakly polarized dielectric, does not appear in the equations of linear theory, Eqs. (16), (17). We will define the field perturbation outside the region containing space charge and consider the continuity of the tangential component of field intensity and the normal component of the induction. We then obtain boundary conditions for E'_x:

$$\frac{dE'_x(1)}{dy} + kE'_x(1) = 0; \tag{18}$$

$$\frac{dE'_x(-1)}{dy} - kE'_x(-1) = 0, \tag{19}$$

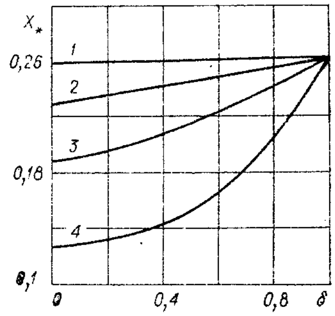

Fig. 2

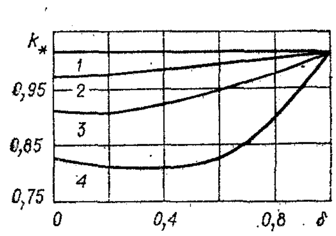

Fig. 3

for the case where the dielectric permittivity of the channel walls coincides with that of the fluid.

Two other boundary conditions for the field are found from the requirement that the normal component of the current must vanish on the fluid–wall boundary. These conditions have the form

$$\frac{1}{\mathrm{Re}_i}\left(\frac{d^2}{dy^2}-k^2\right)\frac{dE_x'}{dy}-E_y\left(\frac{d^2}{dy^2}-k^2\right)E_x'-\frac{dE_y}{dy}\frac{dE_x'}{dy}=0 \text{ at } y=\pm 1. \tag{20}$$

The problem of stability study has now been reduced to analysis of the eigenvalues of system (16), (17) with boundary conditions (19), (20) and boundary conditions of adhesion and impermeability for the velocity. This task presents difficulties well known in the theory of hydrodynamic stability of viscous liquid flows. Moreover, the present problem contains a larger number of similarity criteria, and, of special importance, system (16), (17) is of much higher order than the corresponding system of regular hydrodynamics. The problem is also much more complex than its analog in magnetic hydrodynamics at finite magnetic Reynolds number values.

The stability analysis becomes simpler if we assume that the quantity Re_i is large. In this case Eq. (17) can be written as

$$\left(\frac{d^2}{dy^2}-k^2\right)E_x'=-\frac{\mathrm{M}_a}{1+\mathrm{M}_a(U-c)}\frac{d^2E_y}{dy^2}v, \tag{21}$$

and Eq. (16) takes on the form

$$L[v]=-\frac{i\,\mathrm{Eu}_a}{\alpha\,\mathrm{Re}_a}\left(\frac{d^2}{dy^2}-k^2\right)\frac{dE_x'}{dy}. \tag{22}$$

Differentiating Eq. (21), we reduce system (21), (22) to a modified Orr–Sommerfeld equation

$$L[v]=\frac{i\,\mathrm{Eu}_a\mathrm{M}_a}{\alpha\,\mathrm{Re}_a}\frac{d}{dy}\left[\frac{1}{1+\mathrm{M}_a(U-c)}\frac{d^2E_y}{dy^2}v\right]. \tag{23}$$

Finally, if the dimensionless parameter $P = 2\mathrm{Eu}_a\mathrm{M}_a B^3/\mathrm{Re}_a\,\mathrm{Re}_i$ is still small, the right side of Eq. (23) may be neglected. Equation (23) then transforms to an Orr–Sommerfeld equation. The possibility of neglecting the right side of Eq. (23) at a low value of the parameter P was demonstrated in numerical experiment with $\delta = 0.5$, $\mathrm{M}_a = 10^{-3}$, $B = 1$. As long as $P < 0.002$ the critical Reynolds numbers differ from their limiting value at $P = 0$ by not more than 5.5%.

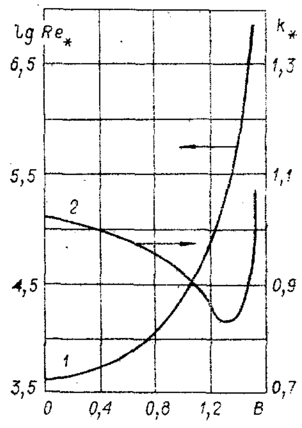

Fig. 4

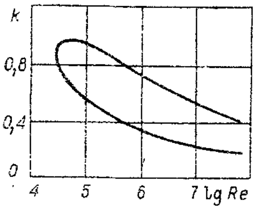

Fig. 5

The electric field creates a definite form in the velocity profile of the basic flow, and thus has a significant effect on its stability. It is this property of the electric field which is dominant in the simplifying assumptions made above. In the general case the direct action of the field on flow velocity pulsations may also prove significant. However, detailed consideration of this electric field interaction mechanism will, generallv speaking. require study of the complete system (16), (17).

In the present study, we have examined the electric field effect on flow stability related to formation of a velocity profile of given form. The problem of eigenvalues for the Orr–Sommerfeld equation with velocity profile (8) was solved using the differential drive method [7]. In accordance with Squire's theorem the original three-dimensional problem was reduced to the equivalent two-dimensional one, so that in the equation $L[v] = 0$ $\alpha = k$.

In Fig. 1 critical Reynolds number Re_* values are shown as functions of the parameter δ (curves 1-4 correspond to B = 0.3, 0.7, 1, 1.3).

The largest Re_* values correspond to purely pondermotor flow ($\delta = 0$). With increase in δ, the values of Re_*, as can be seen from Fig. 1, decrease monotonically to the Re_* value corresponding to Poiseuille flow ($Re_* = 5772$). Marked flow stabilization is attained at $B \gtrsim 1$. At low B the critical Reynolds numbers differ insignificantly from Re_* for Poiseuille flow at all δ. This result could have been predicted earlier, since at low B, velocity profile (8) is practically no different from that of a Poiseuille flow.

Figures 2 and 3 show the dependence of X_* (the critical phase velocity) and k_* (the critical wave number) on δ (curves 1-4 correspond to B = 0.3, 0.7, 1, 1.3). It is characteristic that with increase in the relative contribution of electrical forces to the main flow the quantity X_* decreases.

Of special interest is the ponderomotive flow, produced solely by electrical forces ($\delta = 0$ in Eq. (8)). The dependence of Re_* and k_* on B for that case are shown in Fig. 4 (curves 1, 2). With increase in B the quantity Re_* increases monotonically, and as $B \to \pi/2$, $Re_* \to \infty$. A minimum in k_* is achieved at $B \approx 1.3$. The value of k_* then increases, which corresponds to the character of the problem at B values close to $\pi/2$. At such B values a new characteristic dimension appears – the dimension of the region over which the velocity decreases from values close to unity to zero. The quantity k_* must be inversely proportional to this dimension. Therefore, it could be predicted beforehand that the quantity k_* will increase beginning at B values at which the flow takes on a boundary layer character.

Figure 5 shows an example of the neutral curve for ponderomotive flow at B = 1.

The analysis performed reveals that an external longitudinal electric field stabilizes the plane-parallel flow of a weakly conductive charged fluid. The effectiveness of this stabilization depends on the value of the space charge and the relationship between the components of flow velocity produced by the pressure gradient and ponderomotive forces. With increase in space charge the ponderomotive forces at the channel walls increase. At a sufficiently high electric field intensity, this leads to equalization of the velocity profile in the flow core and an increase in the velocity gradient at the channel walls. Such action of the electric field causes an increase in critical Reynolds number, at least while the transverse electric field induced by the fluid space charge remains sufficiently small.

We note that in the past it has been the stability of equilibrium states in charged liquids which has been studied almost exclusively, and in many cases it has been found that the electric field exerts a destabilizing influence.

The results obtained herein may be used to evaluate changes between laminar and turbulent flow regimes in EHD pumps and to analyze the EHD method of boundary-layer control in a gas.

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EXPERIMENTAL INVESTIGATION OF THE INTERACTION BETWEEN THERMALS

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UDC 532.517.4

Several results of investigations into the interaction between two thermals which are formed during the ascent of initially spherical volumes of a gas which is lighter than the external medium are presented in this paper. It is known that, during their ascent, such thermals are transformed into circular vortices and, moreover, the light gas passes into their toroidal cores [1-4]. The processes involved in the interaction at various different stages during this transformation have been investigated in the work which is being reported.

1. Let us consider the interaction between two thermals. It is assumed that they are formed as the result of the sudden synchronous or nonsynchronous emergence of two equal free spherical volumes with an effective radius R_0, filled with a gas with a density ρ_1 when the density of the external atmosphere is ρ_0. Let L be the distance between the centers of the volumes, τ be the time interval separating the moments when the first and second thermals emerge, H be the height, h be the height at which the thermals merge or at which they actively interact, g be the acceleration of free fall, and $\xi = (\rho_0 - \rho_1)/\rho_0$ be the relative drop in the density. If the effect of viscosity is neglected and it is assumed that the weight deficit $F = Q\xi g\rho_0$ [2], the distance L, and the time interval τ are the main parameters determining the motion being considered, we obtain from dimensional analysis that

$$h^0 = h^0(L^0, \tau^0),\ T^0 = T^0(L^0, \tau^0),\ \alpha = dR/dH; \tag{1.1}$$

$$h^0 = h/R_0,\ T^0 = T\sqrt{\xi g/R_0},\ L^0 = L/R_0, \tag{1.2}$$

$$H^0 = H/R_0,\ \tau^0 = \tau\sqrt{\xi g/R_0},$$

where T is any characteristic time, between the start of the motion and the moment of merging, for example, α is the aperture angle, and R is the radius of the axial periphery of the core of the vortex.

With such an approach, the dimensionless parameters determining the flow are L^0 and τ^0 or just L^0, when $\tau^0 = 0$. In the latter case $h^0 = h^0(L^0)$, $T^0 = T^0(L^0)$. The explicit dependence of the equations of motion and the boundary conditions for the problem on ξ is eliminated here by substitutions of the variables

$$t^0 = t\sqrt{\xi g/R_0},\ x_i^0 = x_i/R_0,$$

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